

The GeoGebra Intelligent m-Tutors

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Overview

- Introduction to Model-Tracing Tutors
- The GeoGebra Intelligent m-Tutors
 - The Domain Expertise Model
 - The Tutoring Expertise Model (Demo)
 - The Student Model
- The MATHESIS LMS
- Further Work
- Discussion

Bloom's 2 Sigma Problem

The Discovery

In 1984 Benjamin Bloom discovered a teaching method for drastically improving educational efficiency, by a factor of two standard deviations, 2σ (2 sigma)

The “average” student within a given class could now perform better than 49 out of every 50 students within a traditional classroom setting.

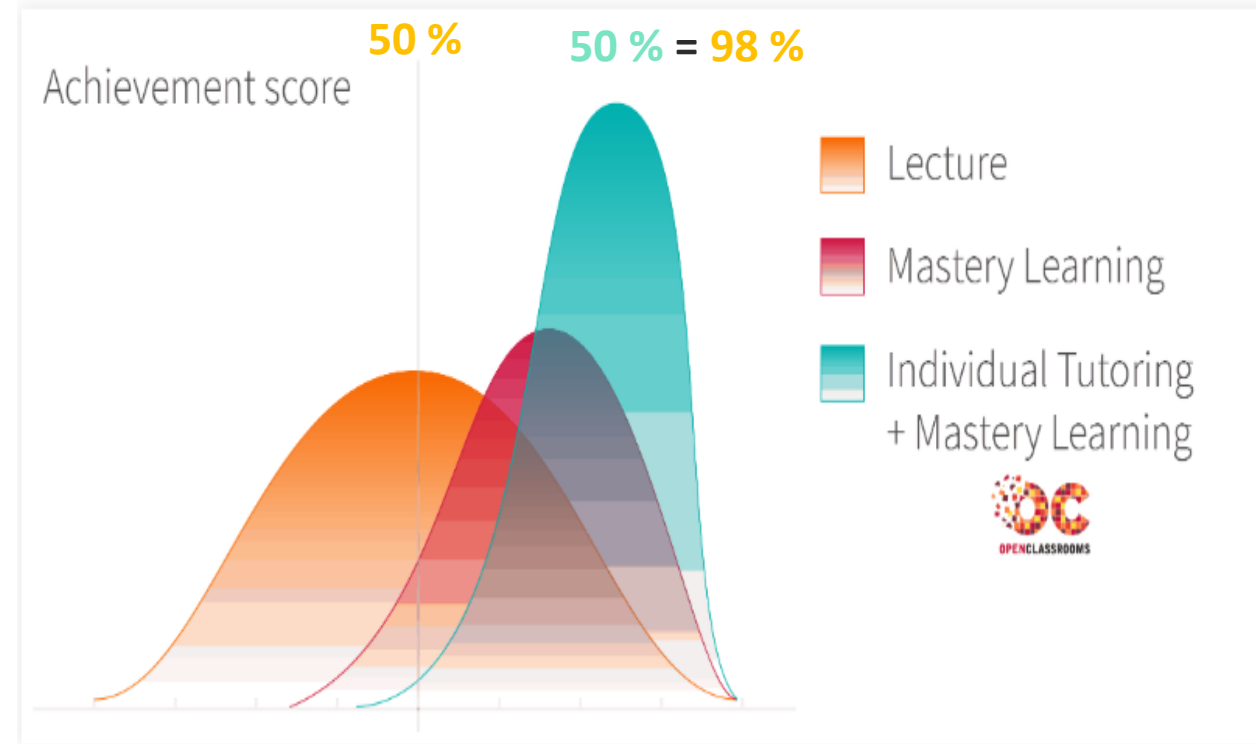
- 1. Mastery learning:** Each student must achieve true mastery of a topic before moving on to the next, more advanced subject.
- 2. One-to-One tutoring:** Each student is provided with a personal tutor who guides him through their learning, suggesting specific exercises and unlocking the individual student's potential on an ongoing basis and ensuring they truly “get” the subject.

Bloom's 2 Sigma Problem

The Unsolved Issue of Resources

Mastery Learning: substantial time would be needed to set up a mastery-oriented teaching framework, and the move away from hour-long teaching blocks would be hugely disruptive to the traditional learning environment.

One-to-One Tutoring: time and cost- intensive, and incredibly difficult to implement for large groups of students (scalability).



<https://www.classcentral.com/report/wp-content/uploads/2016/03/Achievement-3rd.png>

The problem of implementing educational environments as effective as mastery learning + individual tutoring was termed by Bloom as “the 2 Sigma problem”.

Intelligent Tutoring Systems

A Solution to the 2 Sigma Problem

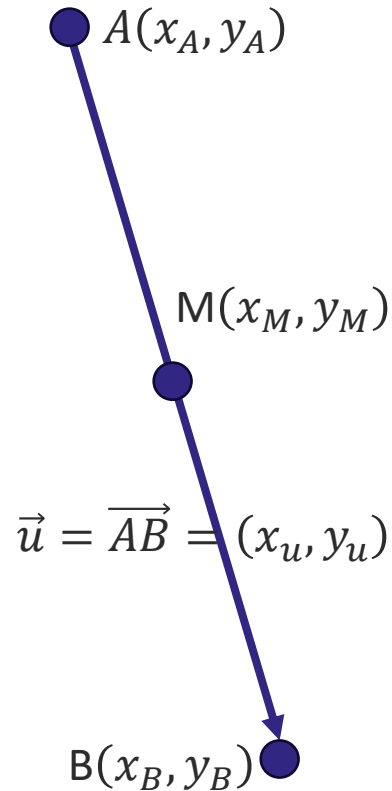
Intelligent Tutoring Systems (ITS), and particularly **Model-Tracing Tutors** (MTT), have been proven quite successful in various STEM disciplines (mathematics, physics, programming). Their success lies in three expertise models:

- The ***domain expertise model*** or ***problem solver***, which represents the problem-solving knowledge of the tutored domain.
- The ***pedagogical or tutoring model*** which represents the teaching knowledge of the system. It guides the student *within* exercises (**One-to-One tutoring**) and *between* exercises (**mastery learning**).
- The ***student model***, which represents the student's mastery level of the domain's cognitive skills (*competences*). Used by the tutoring model to control mastery learning.

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The Domain Expertise Model

- Calculation of vector coordinates from its initial and terminal points: $x_u = x_B - x_A$, $y_u = y_B - y_A$
- Calculation of vector magnitude from its coordinates: $|\vec{u}| = \sqrt{x_u^2 + y_u^2}$
- Calculation of the distance of two points from their coordinates: $(AB) = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$
- Calculation of a segment's midpoint from the coordinates of its endpoints: $x_M = \frac{x_A + x_B}{2}$, $y_M = \frac{y_A + y_B}{2}$
- Investigation of whether two vectors are parallel/vertical or not
$$\overrightarrow{u_1} = (x_1, y_1) // u_2 = (x_2, y_2) \Leftrightarrow \frac{y_1}{x_1} = \frac{y_2}{x_2} \Leftrightarrow x_1 y_2 - x_2 y_1 = 0 \Leftrightarrow \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = 0$$
- Calculation of a line's equation from its slope and one point: $y - y_A = \lambda(x - x_A)$
- Calculation of a line's equation from two points: $\lambda = \frac{y_B - y_A}{x_B - x_A}$, $y - y_A = \lambda(x - x_A)$



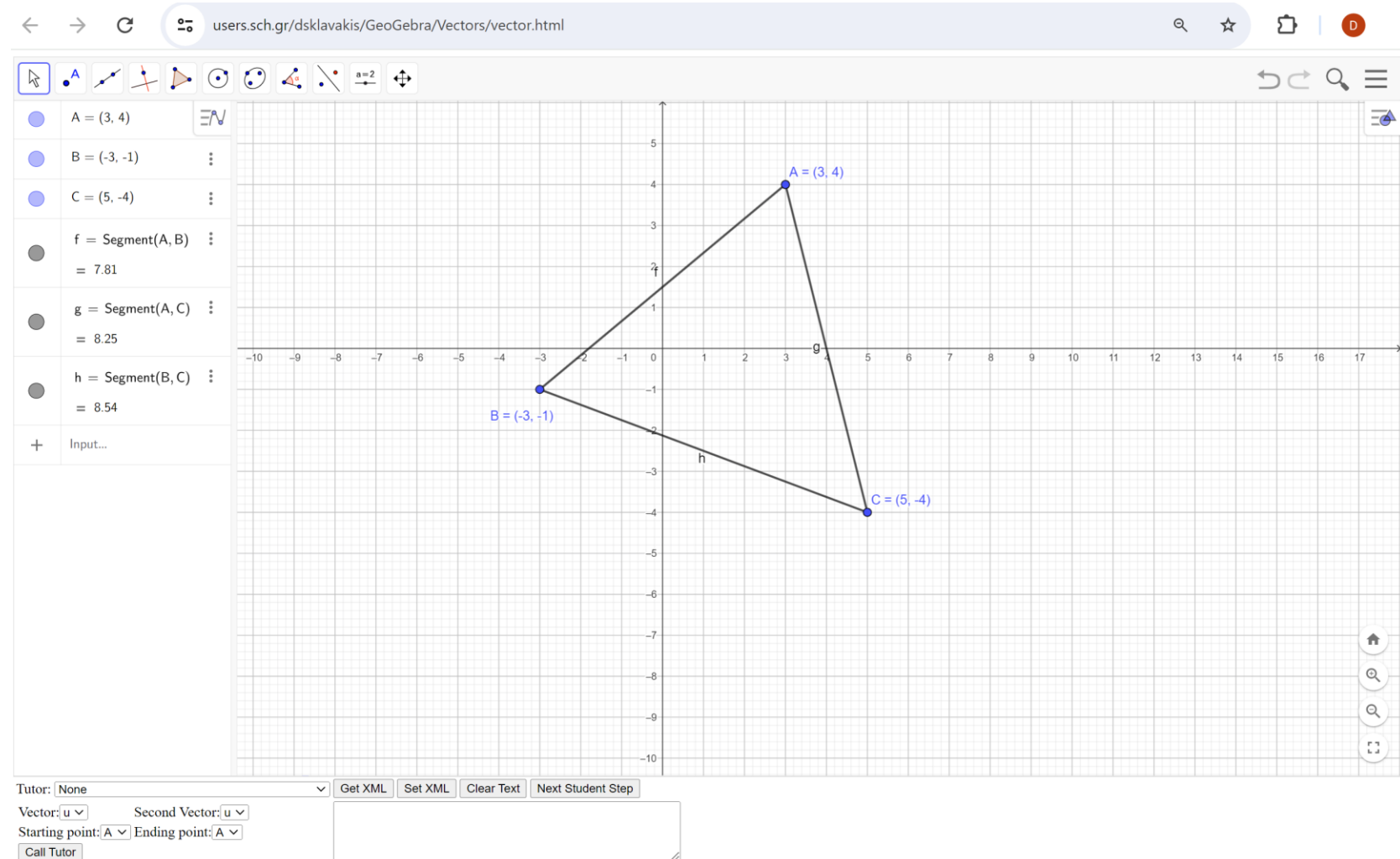
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The Tutoring Model Demo: Guiding the solution of an exercise

https://users.sch.gr/dsklavakis/GeoGebra/MATHESIS_Main_Frameset.htm

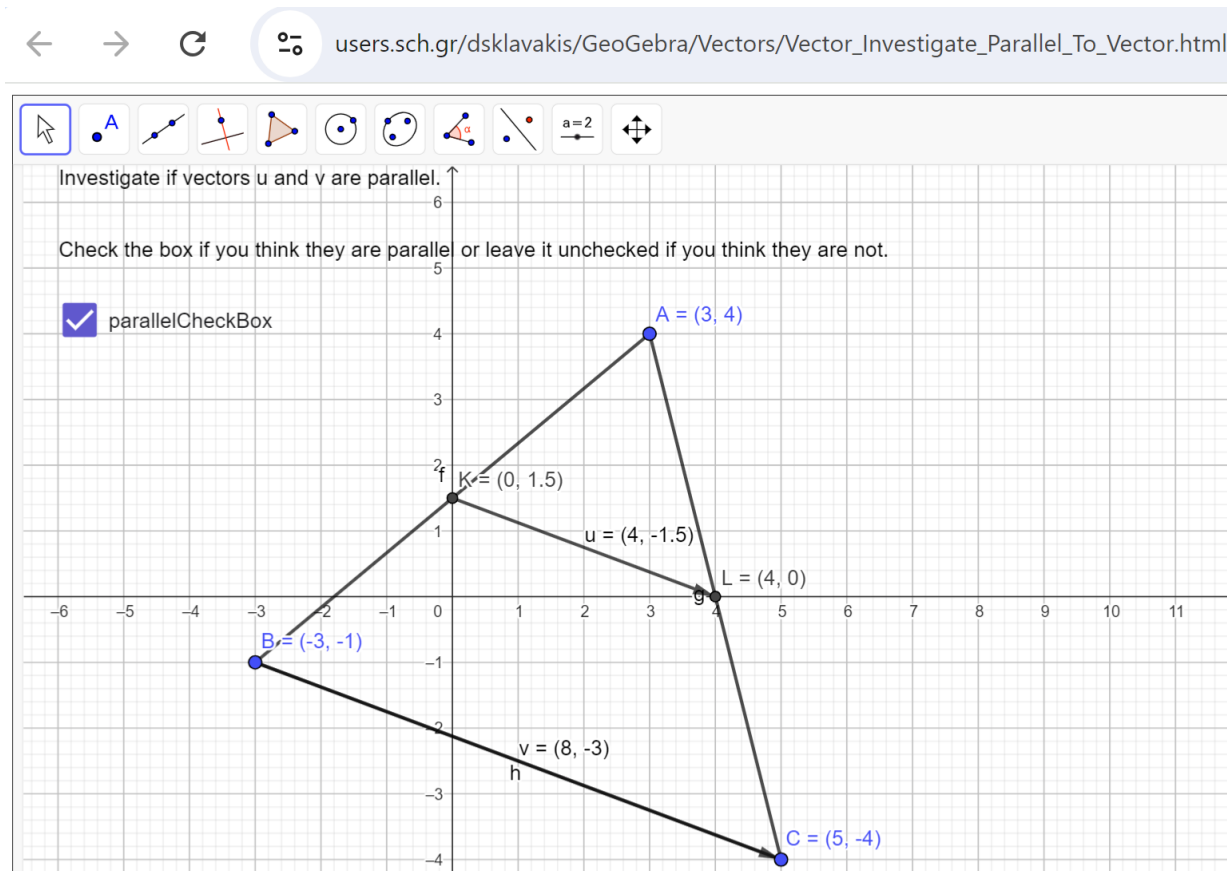
Given the triangle ABC ,
with vertices $A(3,4)$, $B(-3,-1)$
and $C(5,-4)$.

1. Find the midpoints, D and E
of the sides AB and AC correspondingly
2. Show that $\overrightarrow{DE} // \overrightarrow{BC}$

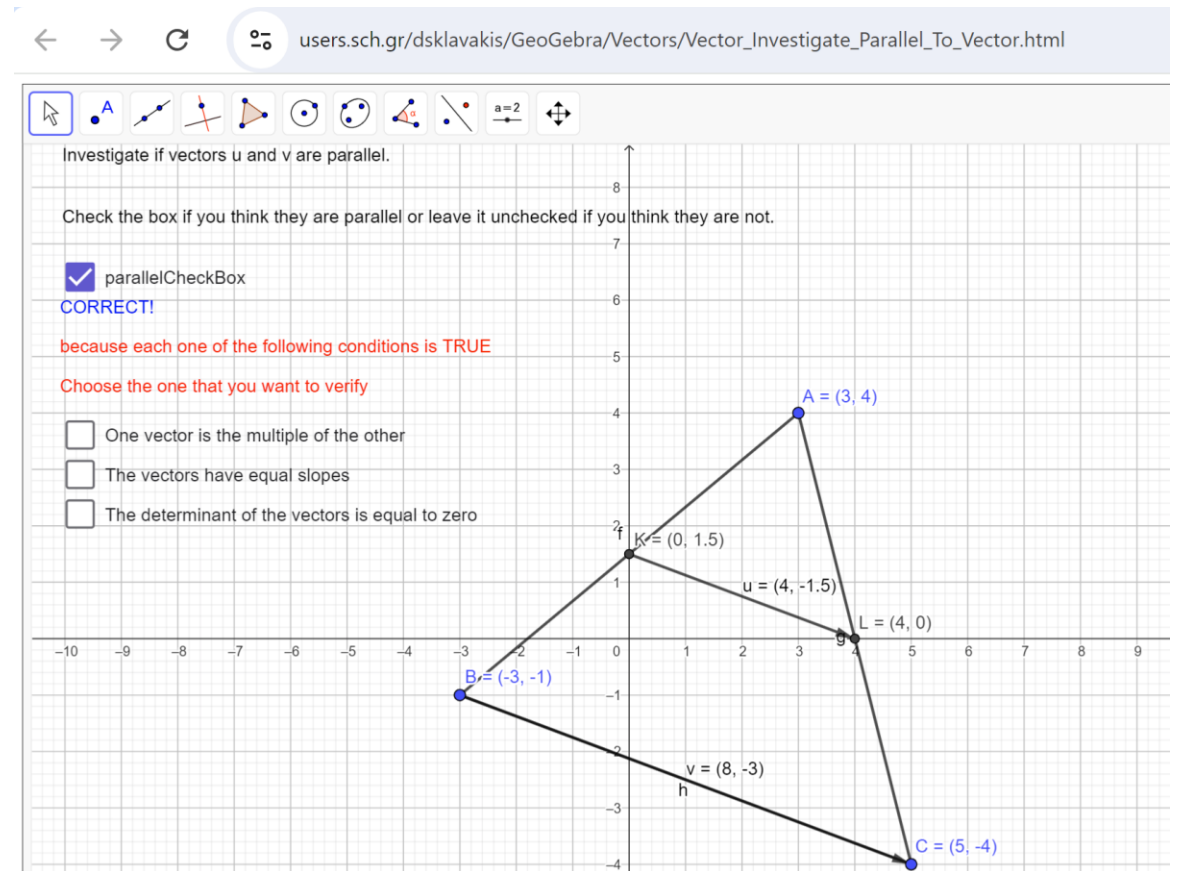


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The Tutoring Model : Deep Model-Tracing (1/3)



1. The student checks the *parallelCheckBox* to indicate that $\vec{u} \parallel \vec{v}$.



2. The m-Tutor gives positive feedback and asks from the student which method was used

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The Tutoring Model : Deep Model-Tracing (2/3)

← → ↺ users.sch.gr/dsklavakis/GeoGebra/Vectors/Vector_Investigate_Parallel_To_Vector.html

Investigate if vectors u and v are parallel.

Check the box if you think they are parallel or leave it unchecked if you think they are not.

☒ parallelCheckBox
CORRECT!

because each one of the following conditions is TRUE

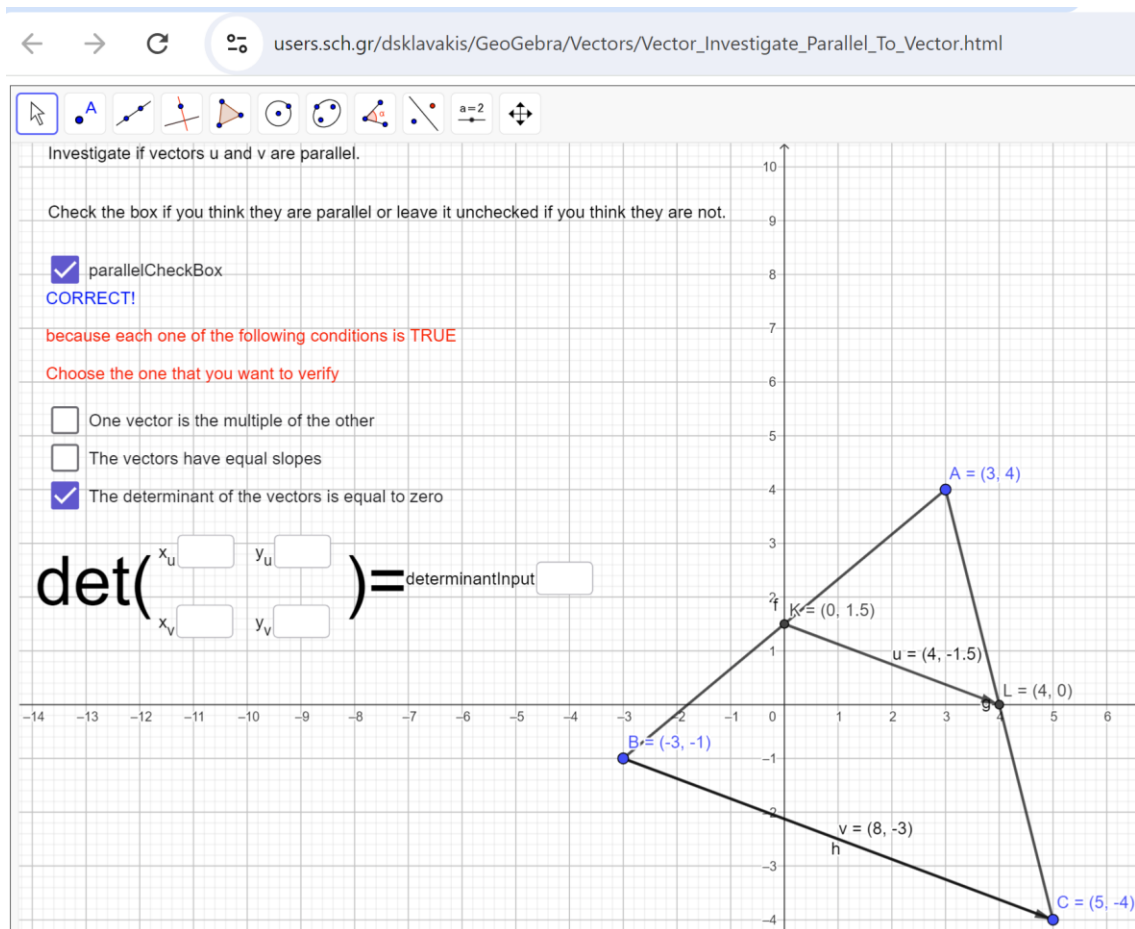
Choose the one that you want to verify

☐ One vector is the multiple of the other

☐ The vectors have equal slopes

☒ The determinant of the vectors is equal to zero

$\det\left(\begin{matrix} x_u & y_u \\ x_v & y_v \end{matrix}\right) = \text{determinantInput}$



3. The students selects the *zero-determinant* method, and the m-Tutor asks to fill in the coordinates of the vectors and the calculated value of the determinant.

← → ↺ users.sch.gr/dsklavakis/GeoGebra/Vectors/Vector_Investigate_Parallel_To_Vector.html

Investigate if vectors u and v are parallel.

Check the box if you think they are parallel or leave it unchecked if you think they are not.

☒ parallelCheckBox
CORRECT!

because each one of the following conditions is TRUE

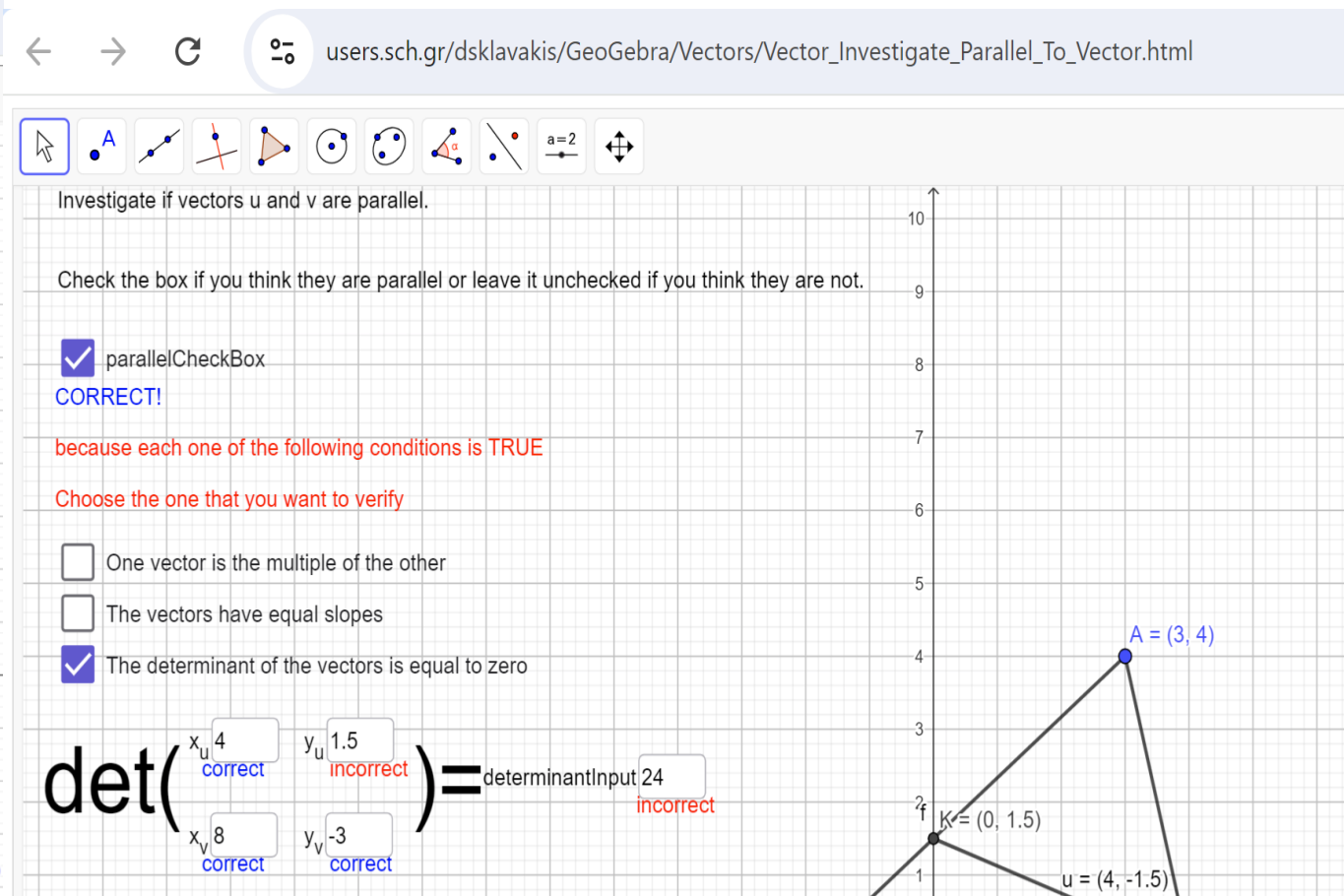
Choose the one that you want to verify

☐ One vector is the multiple of the other

☐ The vectors have equal slopes

☒ The determinant of the vectors is equal to zero

$\det\left(\begin{matrix} x_u & y_u \\ x_v & y_v \end{matrix}\right) = \text{determinantInput}$



4. The student makes two mistakes, indicated by the m-Tutor by the red colour.

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The Tutoring Model : Deep Model-Tracing (3/3)

← → ↺ 📄 users.sch.gr/dsklavakis/GeoGebra/Vectors/Vector_Investigate_Parallel_To_Vector.html

Investigate if vectors u and v are parallel.

Check the box if you think they are parallel or leave it unchecked if you think they are not.

☒ parallelCheckBox
CORRECT!

because each one of the following conditions is TRUE

Choose the one that you want to verify

☐ One vector is the multiple of the other

☐ The vectors have equal slopes

☒ The determinant of the vectors is equal to zero

$\det\left(\begin{matrix} x_u & y_u \\ x_v & y_v \end{matrix}\right) = \det\left(\begin{matrix} 4 & -1.5 \\ 8 & -3 \end{matrix}\right) =$

$\begin{vmatrix} 4 & -1.5 \\ 8 & -3 \end{vmatrix} = \square * \square - \square * \square$

=determinantInput 24
incorrect

5. The student enters correctly the vectors' coordinates but the calculation of the determinant is still wrong.

6. The m-Tutor backtracks even deeper and asks from the student to show how the determinant was actually calculated.

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The Student Model: Competence-Based Records

<u>CompetenceID</u>	Date	Given	Answer	Correct
3 (Segment mid-point)	28/8/2024	<u>A</u> (3,4), B(-3,-1)	(0,3)	0
3	28/8/2024	<u>A</u> (3,4), B(-3,-1)	(0, 0.5)	0
3	28/8/2024	<u>A</u> (3,4), B(-3,-1)	(0, 1.5)	1
6 (Parallel vectors)	28/8/2024	<u>u</u> =(4, 1.5), <u>v</u> =(-8,-3)	parallel	1
5 (Vector length)	28/8/2024	<u>K</u> (0,1.5), L(4,0)	18.25	0
5	28/8/2024	<u>K</u> (0,1.5), L(4,0)	4.27	1

Student competence performance records for formative and summative assessment

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The Student Model: Solution Steps Records

Student Action	JavaScript command for “Playback”
Student selects the m-Tutor <i>Segment Calculate Midpoint From Start End</i> from the <i>Tutor</i> drop-down menu	<code>document.getElementById("tutor").value="Segment_Calculate_Midpoint_From_Start_End"</code>
Student selects point A from the <i>Starting point</i> drop-down menu	<code>document.getElementById("starting-PointsList").value="A"</code>
Student enters 3 as the value of the <i>xCoordinateInput</i> input box for the x-coordinate of the midpoint of segment AB	<code>ggbApplet.setValue("xCoordinateInputVar",3)</code>
Student clicks the <i>Check My Answer</i> button	<code>checkStudentAnswer()</code>

Student actions and JavaScript commands for solution playback

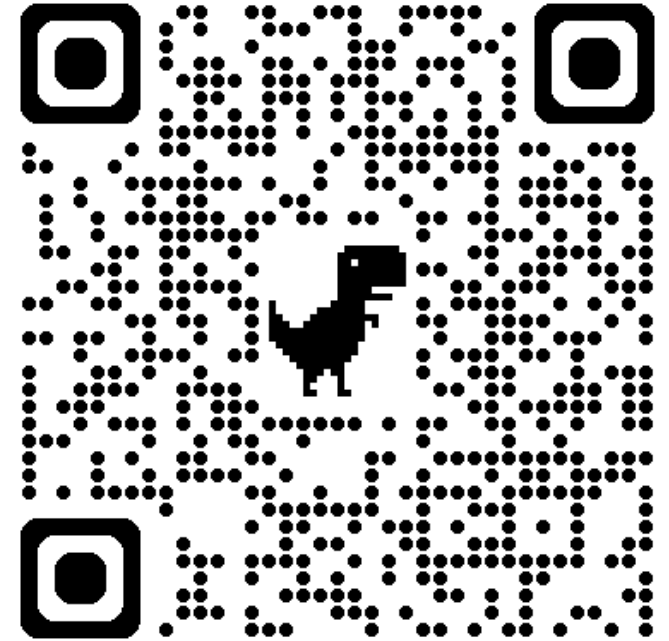
The MATHESIS Learning Management System

Overview

- m-Tutors can be used independently and as stand-alone web-pages,
- MATHESIS integrated Learning Management System (LMS)

https://users.sch.gr/dsklavakis/GeoGebra/MATHESIS_Main_Frameset.htm

- Student/Teacher management: Sign Up / Sign In
- Class management: Create, Insert Student, Delete Student
- Booklet management: Create, Insert, Annotate, Delete exercises
- Assignment of exercises to students
- Playback and Assessment of student assignments both for teachers and students



The MATHESIS Learning Management System

Demo

https://users.sch.gr/dsklavakis/GeoGebra/MATHESIS_Main_Frameset.htm

←

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↺

🔍

users.sch.gr/dsklavakis/GeoGebra/MATHESIS_Main_Frameset.htm

MATHESIS - Intelligent GeoGebra Tutoring School

User

Name

If you are already registered, enter your Username and Password and click the 'Sign In' button.

Username

Password

Sign In

Cancel

If you are not registered, enter your data and click the 'Sign Up' button.

Username

Password

Repeat Password

Fullname

Property

Student ▾

Sign Up

Cancel

Sign In / Sign Up

MATHESIS - Classes Management

Open

Create

Save

Close

School	Grade	Class
EEB2	S5	S5 MA4 ENA

	SN	Username	Fullname
<input type="checkbox"/>	1	fontanmr	FONTAN IRELAND Maria
<input type="checkbox"/>	2	obrienjo	OBRIEN Joshua
<input type="checkbox"/>	3	riderja	RIDER James
<input type="checkbox"/>	4	shielsda	SHIELS Daniel

Add Student

 with Username:

Delete Selected Students

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Further Work

1. Extend the Domain Expertise Model

- Calculation of the point of intersection of two lines
- Calculation of a line's equation parallel/vertical to another line
- Calculation of the acute angle of two lines
- Calculation of the distance of a point from a line;
- Calculation of the projection of a point to a line
- Calculation of the distance of two parallel lines.

2. Authoring Tools for Solution Plans of the top-level competences

- The student selects the top-level competence to perform and the order of performance
- The Authoring Tools will allow Tutors to construct a Semantic Diagram that will define the correct/acceptable order of the competences to be performed

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End of Presentation

Thank you!

Questions?